The Transit System, 1977: Performance, Plans and Potential [and Discussion]

H. D. Black, N. A. G. Leppard, J. C. Blankenburgh, P. G. Sluiter and D. G. King-Hele

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The Transit System, 1977: performance, plans and potential

BY H. D. BLACK

The Johns Hopkins University, Applied Physics Laboratory, Johns Hopkins Road,
Laurel, Maryland 20810, *U.S.A.*

The precision of the Transit System is described since the introduction of the software improvements in 1975. The surveyor's rather than the navigator's use of Transit is emphasized.

The differences between the basic elements of (1) the classical surveying technique, and (2) the Doppler technique are contrasted.

It is necessary to consider the geopotential model as part of the Doppler datum definition. Geopotential model changes create positional shifts which vary from point to point. Datum transformations on a global scale, then, are much more complicated than the recent literature indicates.

Current techniques for minimizing correlated errors, principally drag, are discussed, as are immediate extensions to sub-metre precision.

A major software change in the Transit orbit generation computation is planned for late 1979. A new satellite is currently in limited production. Implications for users of the system are described.

INTRODUCTION

The Navy Navigation Satellite System (Transit) provides a global basis for position information both for navigation and for surveying.

Transit has been available for worldwide use since 1967. The five satellites currently in orbit were all designed in 1963. The newest satellite has been in orbit for 5 years and the oldest for 12 years. There are currently 12 in storage awaiting the failure of those in orbit. A new series of satellites (Nova) is currently in limited production.

The system has been repeatedly described as it has evolved (Guier & Weiffenbach 1958, 1960; Kershner & Newton 1962; Newton 1966, 1967; Pisacane *et al.* 1973; Black *et al.* 1976). This last reference contains an annotated bibliography.

The current performance of the system is reviewed and potential improvements are discussed.

Navigation and surveying

The use of Transit for surveying rather than navigation will be emphasized. Navigation is position determination (latitude and longitude) aboard a moving vehicle (ship), whereas surveying is establishing the position of an Earth-fixed point.

In positioning a ship, a navigator is unconcerned with errors comparable to the length of his ship and, consequently, can use approximations that are unacceptable to a surveyor. Moreover, the surveyor is interested in his position in three-dimensional space, whereas the navigator is only concerned with his latitude and longitude. The different requirements of each should not, however, be unduly stressed: if the navigator's requirements are for extreme accuracy, as in positioning a drilling site for an offshore well, then his needs merge with those of the surveyor.

Local and global datums

Until the advent of artificial satellites, geodesists were hampered by oceans, national boundaries, and rough terrain. Moreover, they were limited in making measurements in the vertical by being denied access to the Earth's centre. In recognizing these obstacles, so-called local datums were defined to achieve high precision within countries or continents (mathematical 'neighbourhoods') (Jones 1973; Mueller 1974; Moritz 1974). Local datums can be characterized by:

(1) A reference point, i.e. a point having defined coordinates.

(2) The defined azimuth of a line in the local horizon plane at the reference point. This is equivalent to locating the meridian on the ground.

(3) The 'deflexion of the vertical' at the reference point, two angles that specify the direction of gravity (the gradient of the geopotential including the centripetal term) at the reference point. (The local plumb bob vertical, the deflexions of the vertical and the line-azimuth taken together define the plane containing the poles of the coordinate system.)

(4) A set of constants: the semimajor axis and flattening of an Earth-associated ellipsoid. Consistent with this ellipsoid a 'standard gravity' formula is defined. These are used in computing principal terms in the station radius and gravity magnitude. More importantly, they become a basis of linearization and are used to approximate the 'real thing' in computing small terms (see Mueller 1974).

Several comments are called for: this is only one of many possible ways to characterize a local datum. Before 1960, polar motion was ignored and astronomical measurements given 'infinite' weight (Jones 1973); consequently, correlated errors as large as 10 m probably exist in local datum coordinates.

The local datum is no longer necessary and should be replaced with a global datum: a single, Earth-fixed, geocentric, coordinate system within which all terrestrial points are specified. This will probably not happen by design but will gradually evolve over the next several decades. In fact, the process is currently an ongoing one. Such a coordinate system will suffice to specify position with a precision of a few tens of centimetres. For higher precision on a global scale, the non-rigid character of the Earth must be considered. Newton (1974) has described the necessary steps to construct a coordinate system on a non-rigid Earth.

There are indications that tables of station coordinates and fiducial points will, in the future, lose most of their importance: the development of a satellite system and user equipment that will quickly produce the geocentric coordinates of a point could dramatically change the surveyor's art.

Doppler and traditional techniques

The last 10 years have been a very confusing and unsatisfying time for someone trying to reconcile coordinate positions obtained (*a*) from traditional sources (land surveys) and (*b*) from satellite techniques. It would seem that it should be a simple matter to transform one set of positions into the coordinate system of the other set. The unsuspecting reader proceeds by studying the coordinate definitions (datums) associated with the data sets. He quickly becomes enmeshed in a mass of details (Mueller 1969; Bomford 1971; Heiskanen & Moritz 1967; Jones 1973; Whitten 1974; Baker 1974). In the Doppler system, an understandable definition of the associated Earth-fixed, coordinate system is difficult to find (Anderle 1976).

The reasons for this state of affairs is both human and technical. Part of the reason is that the classical technique and the Doppler technique start from two different sets of primitives. The choice of these was strongly influenced by 'the observables' in each system.

The basic elements (primitives) of the Doppler technique

For Doppler surveying to exist the following basic elements must exist for developing the system:

(1) A number of satellites equipped with Doppler beacons must be available in long-lived orbits, 1000 km altitude being satisfactory. The satellites must be at various orbital inclinations.

(2) A globally distributed set of tracking sites must be available to measure the Doppler shift and relay it to a central computing facility. (Once the system is developed the global network is unnecessary.)

(3) *Approximations* must be available for the following: (*a*) the tracking site coordinates in a (single) Earth-fixed coordinate system; (*b*) positions and velocities of the satellites at specified epochs (orbit initial conditions); (*c*) a geopotential model (see below); (*d*) satellite and station oscillator frequencies; known relative to a common 'standard'; and (*e*) tracking-site clock epochs synchronized relative to a common standard. A convenient way to enforce this synchronization is to put a secondary time standard in the satellite.

(4) Precise values must be available for: (*a*) the speed of light (299792.5 km/sec); (*b*) GM (the gravitational constant times the mass of the Earth: 398600.8 $\mathrm{km^3/s^2}$); (*c*) the motion of the Earth relative to the nearly inertial coordinate system used in describing the satellite motion: if the data span associated with a single set of initial conditions is limited to a day or two, then the sidereal rate of the Earth ($7.29211585 \times 10^{-5}$ rad per mean solar second), the main precessional term of the equinox and the phase of the Earth's rotation suffice; (*d*) the 'drift' rates ($\dot{f}$, the rate of change of the frequency) of all station and satellite clocks and oscillators (only those drifts having correlation times comparable to the pass length need be known; the best way to know them is to design the system oscillators so that these drift rates are negligibly small).

(5) Measuring and modelling techniques for dealing with ionospheric and tropospheric propagation effects.

(6) A set of internally consistent analyses or computer programs for improving the items listed in (3) above. Most difficult are the procedures that compute the satellite position.

(7) The longitude of one station must be defined, as for any terrestrial coordinate system definition.

The precision (and accuracy) of the resulting system depends on the care and attention which are used in designing and building the satellite and in implementing items (2), (4), (5) and (6). For example, the technique is not very useful until the frequency measurements and computations throughout the complex of tracking sites, satellites and computing programs are internally consistent to $10^{-10} f$, where f is the frequency broadcast by the satellite.

The items listed under (3) are revised by using the Doppler data available from the system itself: items (1)–(7) are deliberately capable of producing redundant information to facilitate a 'tightening-up' of the precision.

The definition of the coordinate system is here but it is not explicitly stated. An explanation is called for.

The origin

The origin of the coordinate system is implicit in a constraint applied in the determination of the geopotential. The geopotential is conveniently written (Kaula 1966):

$$U = -\frac{GM}{R}\sum_{n=0}^{\infty}\sum_{m=0}^{n} P_n^m(\cos\theta)\,\frac{[C_n^m\cos m\lambda + S_n^m \sin m\lambda]}{R^n},$$

where R, θ and λ are the spherical coordinates (radius, colatitude, longitude) of a point exterior to the Earth. A (finite) number of empirical coefficients in this expansion can be determined from the Doppler system's data (Guier & Newton 1965; Guier 1966; Anderle & Smith 1967; Yionoulis *et al.* 1972; Lerch *et al.* 1978). There are four coefficients, in addition to the trivial set $S_n^{m=0}$, that need not be determined. It is easy to show, from the integrals defining these four coefficients, that $C_0^0 \equiv 1$ and that C_1^0, C_1^1, S_1^1 are identically zero *if* the coordinate origin coincides with the mass centre of the Earth. In a simultaneous determination of site coordinates, geopotential coefficients and orbit specifications (initial conditions), the conditions $C_1^0 = C_1^1 = S_1^1 = 0$ constrain the coordinate system origin to the mass centre. In this determination, the tracking site coordinates are considered as free points in three-dimensional space, except the longitude of one site which is fixed.

Two other coefficients, C_2^1 and S_2^1 are usually constrained to zero. This in turn equates the two products of inertia I_{xz} and I_{yz} to zero, implying that the z-axis is assumed to be a principal axis of inertia.

This process of simultaneously fitting site coordinates, orbits and geopotential coefficients by least squares (a number of frequency bias terms are simultaneously determined) is analogous – in classical geodesy – to a 'readjustment of the datum'. (W. H. Guier was one of the first (Guier 1963; Guier & Newton 1965; Guier 1965) to understand that such a determination was possible and to carry it out in a *tour de force* of analysis and computer programming.)

It is therefore necessary to include the geopotential model as part of the (Doppler) datum definition. As a result, we gain access to the Earth's centre of mass for the coordinate origin.

The longitude reference

Fixing the geocentric longitude of one site is equivalent to locating the zero-meridian on the ground. The choice was such that the zero-meridian of the Transit System is highly consistent with the convention that the brass strip in the courtyard at the Old Greenwich Observatory is 0° (astronomic) longitude. This brass strip has longitude $5.69'' \pm 0.2''$ W in the Transit coordinate system (Dillon *et al.* 1977). The x-axis lies in the plane of the reference meridian, passes through the coordinate origin and is orthogonal to the z-axis.

The z-axis

The z-axis of the coordinate system is the CIO pole. (I have previously made the mistake of calling the CIO 'the mean pole 1900–1905'. P. Melchior has kindly corrected me and has described (Melchior 1974) the operational changes that made the two points distinct.) This pole is implicit in the station coordinates of the four operational stations. The daily coordinates of the rotational pole are obtained from Bureau International de l'Heure (BIH) Circulars B/C (Pisacane *et al.* 1973).

As we know (Beuglass & Anderle 1972) it is not necessary to use BIH coordinates for the instantaneous rotational pole. These coordinates could be determined from the system, but this is not the current practice.

The y-axis

The y-axis is conventionally defined: lying in the equatorial plane (i.e. passing through the coordinate origin) and normal to the x–z plane. The sense of y is chosen so that (x, y, z) form an orthogonal, right-handed system.

The ellipsoid

Since the ellipsoid definition occupies a central place in a local datum definition, its omission in the satellite case is a striking difference between satellite-derived datums and those that are classically defined. An ellipsoid is here but it is very much in the background. (The system operates quite nicely as a 'surveying instrument' without the use of an ellipsoid.) The ellipsoid is implicit in the geopotential model (Cook 1959; Rapp 1970). The point here is that the ellipsoid is a derived, ancillary entity in the satellite discipline rather than occupying a central place as it does among the elements of classical geodesy. The ellipsoid definition is not used anywhere in the orbit computation. In a Doppler datum readjustment, the ellipsoid is almost the last thing derived, and to obtain it, levelling data obtained on the ground from each of the tracking sites are used. The Doppler-derived station radii, with the station heights above mean sea level subtracted (the 'orthometric' height), are points on a mean sea level equipotential surface, the geoid. An ellipsoid is fitted to these points by least squares.

The navigator (not the surveyor) uses the ellipsoid in computing his distance from the Earth's centre of mass: the sum of (*a*) the radial distance from the centre of the Earth to the ellipsoid; (*b*) the local distance from the ellipsoid to mean sea level, the so-called geoid height, and (*c*) the height of the navigator's antenna above the sea. The geoid height ('undulation') is computed from a first-order series expansion of the geopotential about the ellipsoid reference figure:

$$U(r, \theta, \lambda) = U_{\mathrm{ref}}(r, \theta, \lambda) + \left.\frac{\partial \bar{U}}{\partial r}\right|_{r,\theta,\lambda} \cdot \Delta r,$$

where Δr is the geoid height and U_{ref} is the geopotential of the Earth-associated ellipsoid (Rapp 1970). The geoid height is supplied to the navigator in the form of a map (see Staff of the Space Department 1975). Since 1975 there have been dramatic improvements in the sea surface geoid produced from the Geos-3 altimeter results (figure 9): a 60 cm × 120 cm geoid map giving 1 m contours over most of the oceans is given in Brace (1977).

Relations between various datums

Considering 'the' datum as a set of geopotential coefficients, station coordinates and constants, transformations between various datums are not position-invariant. A relation that is valid in one area may not give 'good' results for another. The reason for this is not hard to find: the change in the datum produces satellite position differences that are geographically correlated. A description of positional differences that arise as a consequence of satellite-derived datum changes is given by Holland *et al.* (1977). Figure 1, taken from that report, shows the along-track difference (the component of the vector difference between the two ephemerides resolved in the along-orbit direction) for four different satellites; one ephemeris computed with Transit station coordinates (and geopotential coefficients) and another ephemeris computed with the corresponding APL

4.5 quantities (Black 1968). All the orbits are polar, have different nodes and perigee arguments, and have (slightly) different eccentricities and mean motions.

The abscissa is time-elapsed since all the trajectories passed through a common geographical point. As the polar orbits have periods that are small fractions of a sidereal day, the large peaks on this figure occur when all satellites are (approximately) at the same longitude. As figure 1 shows, there is appreciable geographical correlation in the difference created by the datum change. Not all of this effect would show up in a site survey. This along-track difference would cause a latitude shift which would reverse sign with the direction of satellite motion. Consequently, there would be a tendency to average out part of this difference in a multi-pass solution. The satellite position difference remaining in the two site surveys (the 'local datum shift') would depend on, for instance, the number of passes used, the number of north-going to south-going passes, other errors in the system, and asymmetries in the geometry between north and south passes. Holland *et al.* (1977) compute the coordinate changes associated with correlated orbit differences. These computations indicate the effect of changing datums when consistency between site coordinates and a geopotential model are an integral part of the datum definition. The question arises whether or not these changes could have been removed with a coordinate rotation and scale change as described below: in changing station coordinates we were careful to preserve the same longitude reference and pole of the coordinate system, i.e. not to rotate the coordinate system. The largest change in any station coordinate was 6.1 m in latitude at Hawaii. GM was decreased by 3 parts in 10^6 (Black 1976).

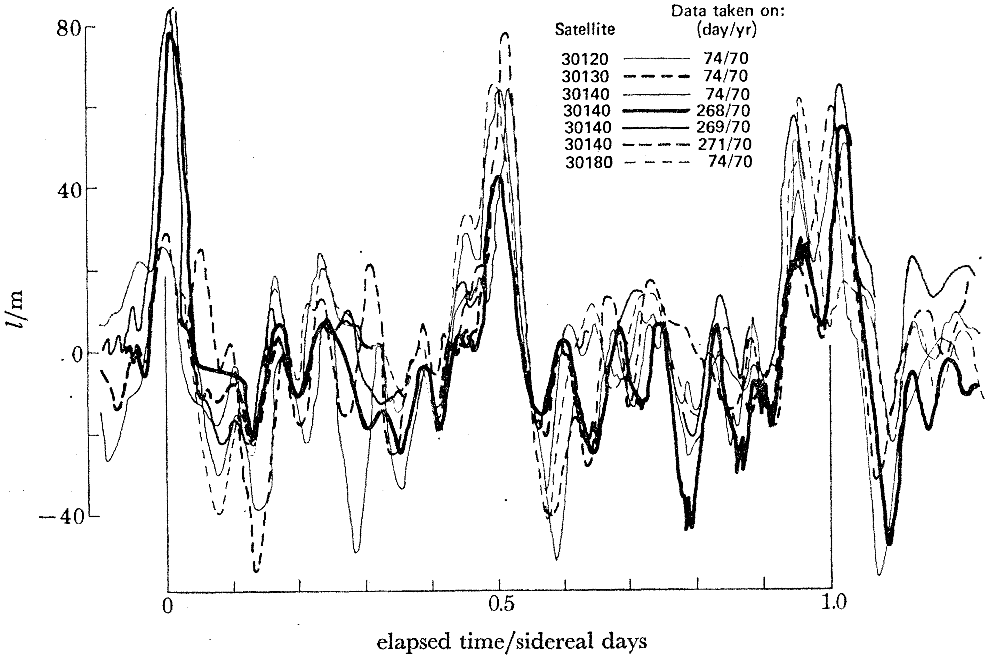

FIGURE 1. Along-track difference due to geodesy change. For a number of different satellite ephemerides, the differences are geographically correlated.

We shall perhaps make changes again, smaller than the current global system precision (3–5 m), when the geopotential models are sufficiently precise to justify changing again. Recent improvements facilitated by Geos-3 altimeter data have produced dramatic results (Brace 1977;

Lerch *et al.* 1978). Within the next year or so, it seems that there will be ample justification to change geopotential coefficients and Transit station coordinates, i.e. to readjust the Transit system datum.

In comparing satellite-derived positions with those from classical adjustments, Lambeck (1971) transforms three local datums to a global geocentric system. He derives the seven-element transformation (three rotations and three translocations plus a scale change) from station co-ordinate differences at common points.

CORRELATED ERRORS

The error budget, like the system, has remained unchanged for several years (Black 1976). Table 1 shows this budget, and until items 3 and 4 are appreciably reduced, there is hardly any point in discussing the others. We will have more to say on this later.

TABLE 1. TRANSIT SYSTEM: SURVEYOR'S SINGLE-PASS ERROR BUDGET (OCTOBER 1977)

	error/m
(1) uncorrected propagation effects (3rd-order ionospheric and neglected tropospheric effects)	1–5
(2) instrumentation (navigator satellite oscillator phase jitter)	1–6
(3) geodesy (uncertainty in the geopotential model)	5–10
(4) incorrectly modelled surface forces (secular error growth due to incorrect period; drag and radiation pressure)	10–25
(5) unmodelled U.T.1 – U.T.C. effects and incorrect coordinates of the pole	1
(6) ephemeris rounding error (last digit of ephemeris is rounded)	5
r.s.s.	12–28

Drag

We should qualify the statement that the error budget has not changed; the 11 year solar cycle is currently approaching a peak in 1980 (Schatten 1978). There will be an occasional day when the drag model (*C.I.R.A.* 1972) poorly represents the upper atmospheric air density. We have recently seen one such day after a solar storm on day 97 (7 April) 1978 (see figure 2). We know enough now to prevent this from happening again (we believe): the orbit plane of that particular satellite intersected the maximum-density region which is at 1700 h local time (5 h ahead of the Sun in right ascension) (Jacchia 1977).

We have made appreciable progress in understanding one source of drag bias: there is a semi-annual term in the atmospheric air density (Cook & Scott 1969) which we found to require modification (Eisner & Yionoulis 1977). However, a model representing current knowledge of the upper atmospheric air density requires constant attention and change. The usual manifestation of incorrectly modelled drag is a linearly (actually a weak quadratic) growing error in the along-orbit, 'along-track', position of the satellite. This error is easy to evaluate *post facto*: by using several days of Doppler data an ephemeris is generated and extrapolated (numerically integrated) beyond the epoch of the last data point, predicting the position into a future which is past. Using data acquired at known sites we can evaluate the satellite position error by navigating the site positions, a computation having a known *a priori* result. The error in the computation

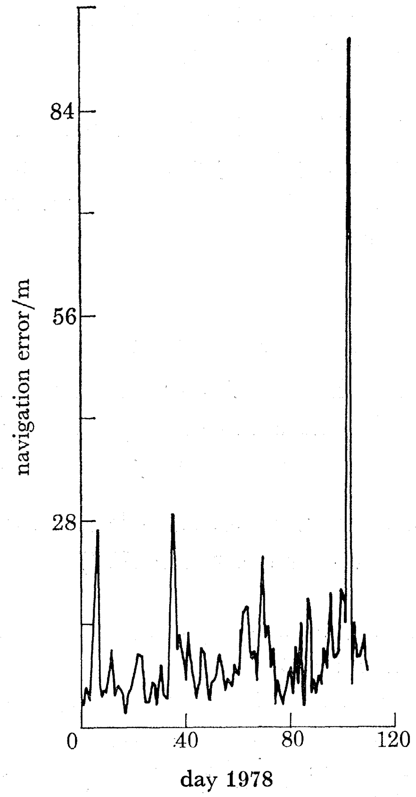

FIGURE 2. Navigation error: satellite 30140. On 7 April 1978 (day 97) a large solar flare occurred which dramatically affected the upper atmospheric air density.

then becomes a measure of the satellite position error. The along-track component of such a computation is shown in figure 3. Here the orbit error clearly grows linearly with time. The sign convention is such that the negative slope corresponds to a modelled drag less than the actual drag. The slope is negative about as often as it is positive but superimposed on a long-period term mentioned above. Figure 4 shows values of this slope for satellite 30140 during 1977.

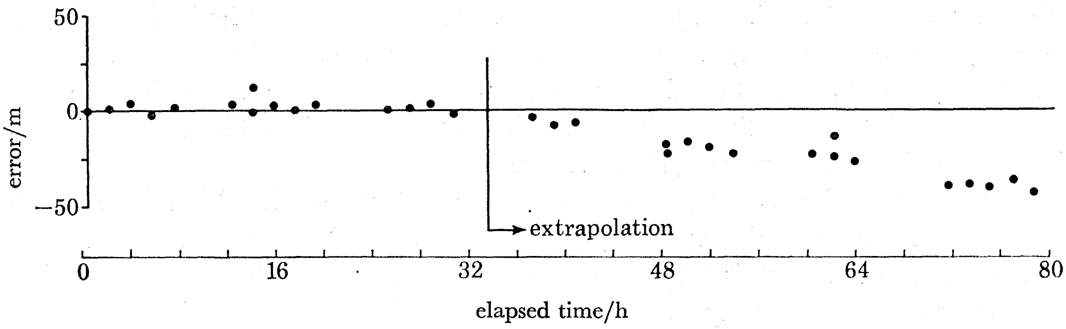

FIGURE 3. Along-track component of orbit error. The error, when the ephemeris is extended outside the data interval, grows owing to the effects of drag. Time origin: 1977 day 338, 6 h 00 min 00 s.

Eisner & Yionoulis (1977) show that the error in the modelled mean air density ($\delta\rho$) is proportional to this slope:

$$\delta\rho = -\frac{2}{C_d}\left(\frac{mn}{3AV}\right)\frac{A_1}{V\Delta t},$$

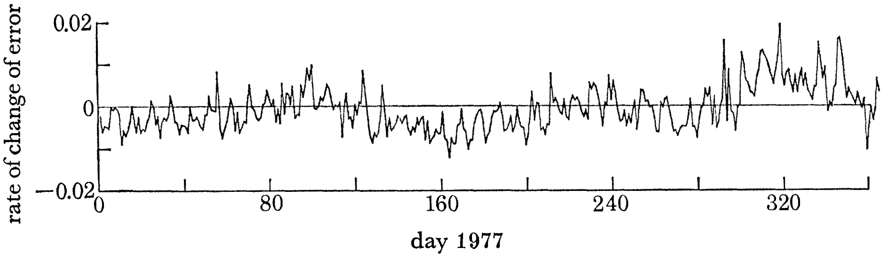

FIGURE 4. Time rate of change of the along-track error: satellite 30140. A long period term is quite apparent.

where C_d is the drag coefficient, m is the satellite mass, n is the mean motion, A is the frontal area of the satellite, V is the speed of the satellite, Δt is the data span used in deriving the orbit specification, and A_1 is the slope of the along-track error growth in metres per unit change in the mean anomaly. This equation is the principal analytical tool used in revisiting the air density model with the along-track error growth as a source of data. The term in parentheses is about 2×10^{-9} g/cm^3.

Removing the effects of correlated errors

Figure 3 shows that the drag-induced errors for a given satellite are strongly correlated over the extrapolation interval. The fact that the errors are correlated for appreciable lengths of time provides the basis for a number of techniques for removing or minimizing their effects.

(1) If two or more points ($\bar{r}_0$, $\bar{r}_1$, ...) are being surveyed and if a satellite can be simultaneously observed from several points, then their relative positions ($\bar{r}_1 - \bar{r}_0$, for example) are more accurately produced than is either absolutely. Because of the correlation, a large part of the error is eliminated in the subtraction. This technique is called 'translocation' (Westerfield & Worsley 1966; Kouba & Wells 1976; Wells 1976).

(2) Including equal numbers of north-going and south-going passes of a given satellite tends to cancel some of the drag-induced latitude errors.

(3) A logical extension of the translocation technique is to include models for satellite-associated errors as part of a multi-station survey. For example, the drag error in the ephemeris is modelled as a linear function of time; the slope and intercept are then determined along with the survey coordinates. Brown (1976*a*, *b*) has developed and extended this 'short-arc technique' and reports sub-metre accuracy for regional results. See Kouba & Wells (1976) for a different implementation of this idea.

TROPOSPHERIC REFRACTION

The detailed tropospheric effects on Doppler surveying and navigation are now basically understood (Hopfield 1969, 1971): the troposphere is not dispersive, its index of refraction is not frequency-dependent, but rather it depends on the pressure, temperature and water vapour pressure (Smith & Weintraub 1953). As a consequence, correction for the tropospheric effect relies on modelling and weather data. The troposphere increases the apparent range to the satellite. Because of the form of the index of refraction, it is convenient to separate the correction into the sum of two corrections, one due to the dry atmosphere and another due to the 'wet', the water vapour. Below 5° elevation, there is an additional 'curvature' effect. The dry term accounts for 85–90% of the combined effect. Figure 5 shows the dry term for three different

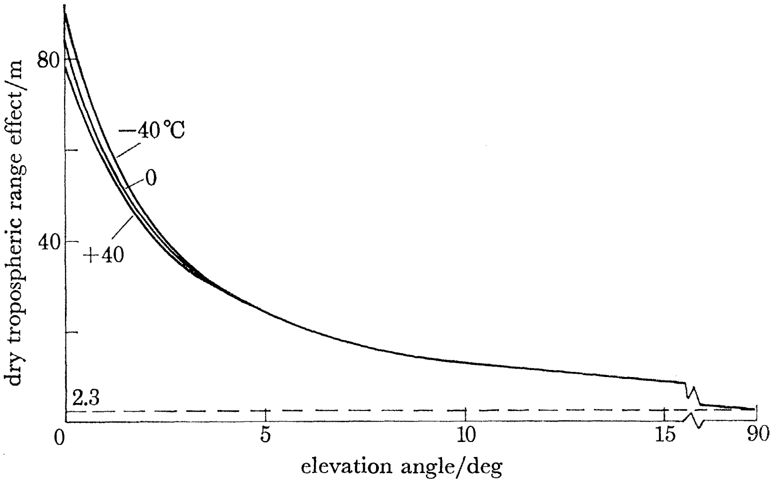

FIGURE 5. Effect of the dry troposphere on the electromagnetically measured range. The zenith effect is 2.3 m. Water vapour in the atmosphere augments the dry tropospheric effect by 10–15%. In preparing the figure, a lapse rate of 6.7 K/km was used.

surface temperatures. By current standards, the effect is too large to ignore; moreover, if ignored it will force a correlated error into the longitude and height (Black 1976).

For elevation angles greater than 5°, the dry term can be written (in metres)

$$\Delta s_{\mathrm{d}} = 2.343\,P\left(\frac{T-4.12}{T}\right)I_{\mathrm{a}};$$

$$I_{\mathrm{a}} \approx \left[1-\left(\frac{\cos E}{1+0.15\,h/r_{\mathrm{s}}}\right)^{2}\right]^{-\frac{1}{2}},$$

where P is the surface pressure at the site in standard atmospheres ($1\ \mathrm{atm} = 1013.25 \times 10^{2}\ \mathrm{Pa}$), T is the surface temperature in kelvins, $h = 148.98\,(T-4.12)$ is the effective 'extent' or 'height' of the dry troposphere in metres, E is the instantaneous elevation angle of the satellite, r_{s} is the distance in metres from the centre of the Earth to the site, and Δs_{d} is the tropospheric correction in metres (Black 1978).

The simple approximation (for elevation angles above 30°) to the above equation,

$$\Delta s_{\mathrm{d}} = 2.31\,P \operatorname{cosec} E,$$

is practically exact but between 6 and 30° the approximation is 13% in error, which is too large to be useful for modelling but suggestive of the important functional dependence.

The theory for the wet term is in a much less satisfactory state, principally because we cannot describe the height distribution of water vapour with appreciable accuracy. Hopfield (1969) has found that the analytical forms which have been derived for the dry term have appreciable validity for the wet. Moreover, since the wet effect is about an order of magnitude less than the dry, we do not currently have to be so careful. Hopfield's finding, together with the intuitive appeal of the cosecant approximation, suggests that we write for the sum of the wet and dry terms

$$\Delta s = C_{\mathrm{T}}\,I_{\mathrm{a}},$$

and determine C_{T} as a pass-associated bias. This removes a correlated error most of the time, even without constraining the uncertainty in C_{T}. Figure 6 is taken from Eisner (1977): the top figure shows the residuals from a (along-track, range, frequency-bias) 'navigation'. The bottom

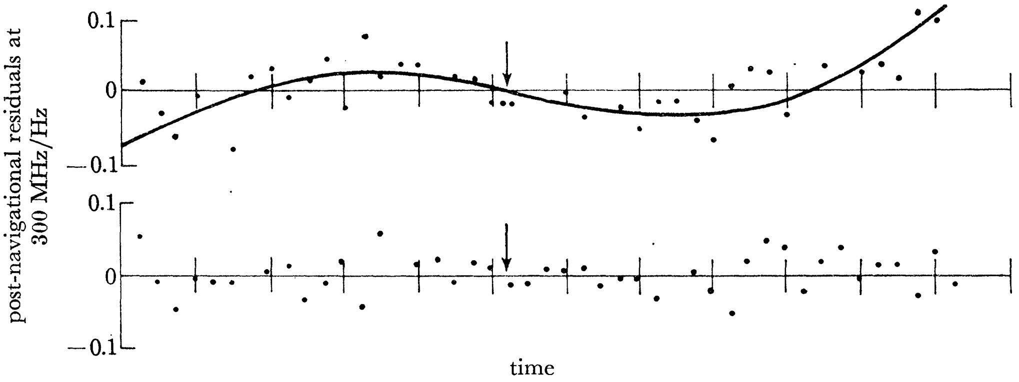

FIGURE 6. Effect of fitting a tropospheric bias on range-difference residuals. The tropospheric biases are clearly present on both ends of the top figure (r.m.s. = 0.03888 Hz). Fitting to a single parameter (lower figure) clearly removes the biases (r.m.s. = 0.02462 Hz). Elevation at time of closest approach (arrows) = 27°.

figure includes additionally a fit to the tropospheric parameter C_T. The reduced correlation in the residuals is quite apparent. As a result of including the tropospheric parameter fit, the site coordinates changed by 9 m for this particular determination.

The key to reliable tropospheric parameter estimation is better low-elevation models where the effect is largest. We have worked intensively in this area (Hopfield 1976*a*, *b*; Goldfinger *et al.* 1978) but there is still much work to do.

Ionospheric refraction

The two-frequency technique for ionospheric compensation (Guier 1961) has its genesis in the work of Seddon (1953), Weekes (1958) and Graves (1960). The index of refraction of the ionosphere, in the frequency region 150–400 MHz, is frequency dependent and as a consequence the Doppler shift can be written for either frequency

$$\Delta f(t) = -\frac{f}{c}\dot{\rho}(t) + \frac{a_1(t)}{f} + \frac{a_2(t)}{f^2} + \frac{a_3(t)}{f^3} + \dots.$$

The coefficients $a_1(t), \dots, a_3(t)$ are integrals of the weighted electron density over the instantaneous line of sight to the satellite. The term $a_1(t)/f$ is called 'first order'. Typical values of the three terms are shown in figure 7 for a satellite transmitter at 50 MHz and moderately elevated ionospheric conditions. Clearly at this frequency, the first and third order terms are about the same order of magnitude and the second order term is negligible. At the lowest Transit frequency (150 MHz) the third order term is typically an order of magnitude less than the first order term and the second order term remains negligible. This observation provides the motivation for truncating the series after the first order term, broadcasting two frequencies and solving a pair of simultaneous equations to eliminate the first order term from further consideration. This leaves part of the third order term as a correlated error (its shape resembles the tropospheric effect but is opposite in sign) in the measurements. This problem has been extensively studied (Tucker 1970). Because of the anti-symmetric shape of the error (figure 7), the resulting error appears in site altitude and site longitude. Figure 8, taken from (Tucker 1970), shows the residual error resulting from uncorrected third order ionospheric effects in a navigation solution. Data acquired while the satellite was below the 'elevation cut-off angle' was not used in the computation. Clearly, because the third order effect is largest at low elevation angles, data

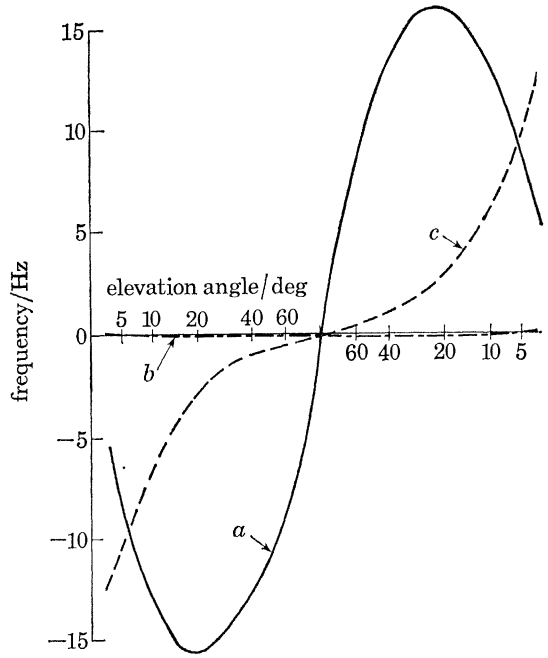

FIGURE 7. First (*a*), second (*b*), and third-order (*c*) ionospheric contributions to the Doppler shift (scaled to 50 MHz). Raising the frequency supresses the third-order term relative to the first. The second-order term is negligible in any case. Spherically symmetric ionospheric model; $N_{\max} = 2 \times 10^{12}$ electrons/m^3 ($f_{\text{crit}} =$ 12.7 MHz); $h_{\max} = 250$ km ($f = 50$ MHz); satellite in circular polar orbit ($E_{\max} = 90°$).

below 5–10° should not be used. The error is largest when the ionosphere is most active which is in turn correlated with the time of day, season of the year and the 11 year solar u.v. activity cycle (Schatten 1978).

We have indicated the dependence on season and solar activity with the two smaller graphs in figure 8. The error is clearly least in spring and summer, almost disappears at night and is currently increasing with the rising solar u.v. activity.

Not all of the error shown in figure 8 appears in the surveyed coordinates: the error is correlated over several hundred kilometres, distances such that the line of sight to the satellite from any two points passes through the same ionospheric structure. Consequently, relative coordinates derived from the 'translocation' or 'short-arc' technique contain only a small fraction of the error. The absence of the error in passes made in the night tends to halve the resulting error in a multi-pass site determination. With all of these precautions, however, it would be difficult to claim sub-metre accuracy for the coordinates of a point in a global datum.

POTENTIAL FOR IMPROVEMENT

In developing and using the Transit System, the question has frequently been asked: what are the accuracy limits of an improved Transit System?

In a long analysis, Newton (1976) has addressed this issue. He finds that the limit on the single-pass fix precision is about 18 cm (on a global scale) plus the error assignable to geopotential model uncertainties. The improvements from using multiple passes (or co-location) would then follow in the usual way. This same limit exists not only for Doppler location but for other types of space-orientated systems. I shall briefly outline the thrust of Newton's analysis.

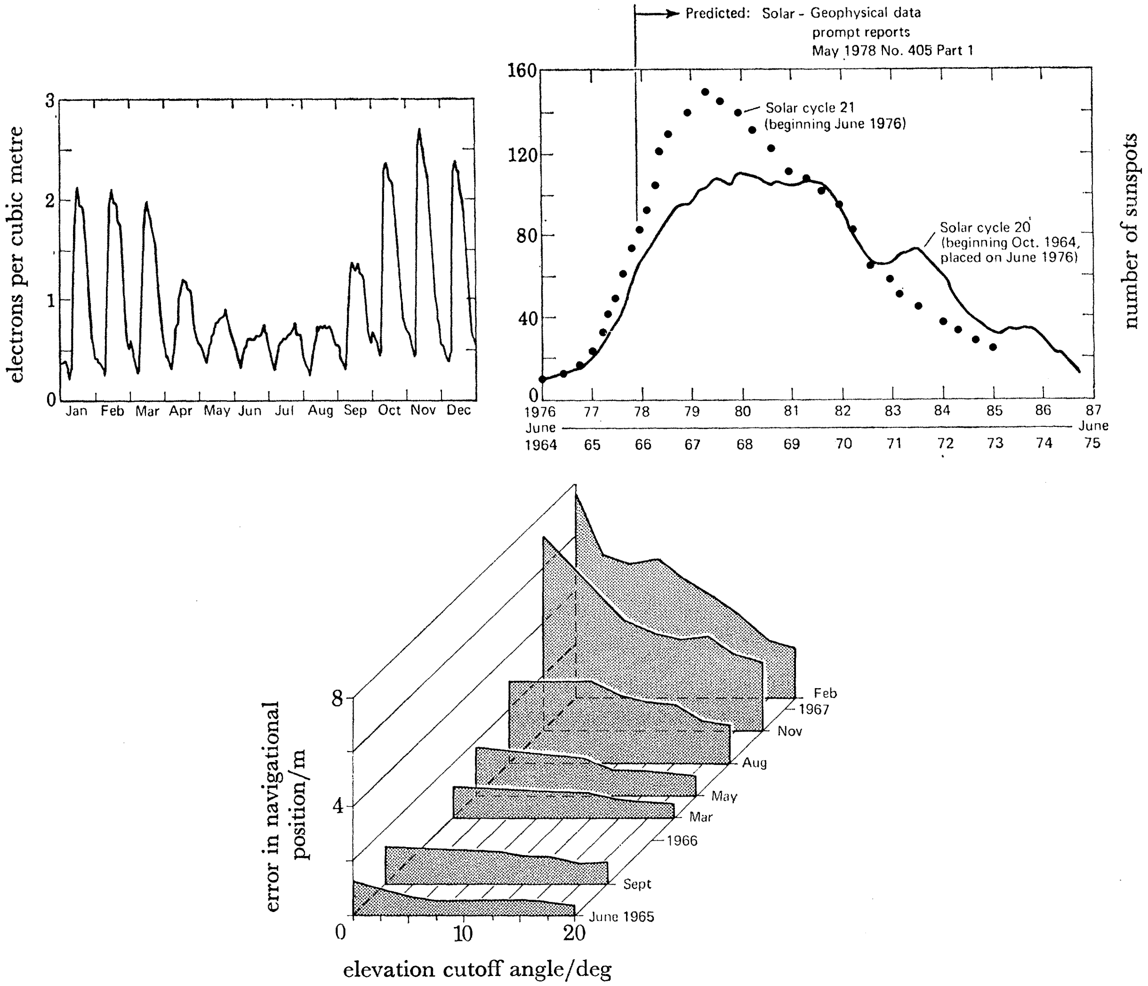

FIGURE 8. Navigation error caused by uncorrected third-order ionospheric refraction on the 150–400 MHz Doppler data. The upper left figure superimposes the diurnal cycle on the seasonal variation: the ionospheric effect is less in summer than in winter and autumn. It nearly disappears at night. The upper right figure indicates that the years 1977–81 will be somewhat more disturbed than were the years 1965–9.

A number of changes would be required which are technically trivial, that is, they require no new knowledge or fundamentally new hardware devices.

(1) The ephemeris of the satellite as stored in the satellite would have to be stored to the nearest 1 cm, rather than to the nearest 10 m.

(2) We should (*a*) raise the frequencies broadcast by the satellites from 150 and 400 MHz to 600 and 1600 MHz, or (*b*) broadcast a third frequency, or (*c*) provide a model for reducing the third order ionospheric error by about two orders of magnitude. Efforts at both the University of Texas (Tucker *et al.* 1976) and the Johns Hopkins University Applied Physics Laboratory have successfully removed about 70% of the third order ionospheric effect by using modelling techniques. It is significant here that the 150 and 400 MHz choice of frequencies was largely determined by the electronic technology of the early 1960s. Raising the frequency of the satellite would in turn require raising the power radiated from the satellite if non-steerable antennae are to be utilized.

(3) The antennae throughout the system need minor improvement. The antenna phase

centres are not fixed (relative to the Earth) but change with satellite elevation angle. D. E. Wells (see Kouba 1976) has measured the phase centre motion and found that for a commonly used receiving antenna (the 'CHU'), the effective (150 and 400 MHz) phase centre of the antenna moves vertically about 25 cm as the satellite crosses the sky. Better antennas could be used or the effects removed by modelling.

(4) The frequency stability of the user and satellite oscillators would have to be improved by about an order of magnitude. The current stability is several parts in 10^{11} both for averaging times of 1 s and for 1 day. Computer simulations (Guier & Weiffenbach 1960) show that a frequency instability of 3 parts in 10^{11} per hour contributes 40 cm to the single-pass fix error.

The crystal oscillator designed for the new series of Transit satellites (Nova) has a frequency stability easily meeting this requirement (Norton 1976). Norton's design achieves a frequency stability of 3 parts in 10^{12} for averaging times of 1 s and 4 parts in 10^{13} for averaging times of 10–100 s.

(5) The satellites and station clocks would have to be slightly improved. The satellite clocks are now consistent with U.T.C. as defined by the U.S. Naval Observatory with an r.m.s. error of about 20 μs while the station clocks in the Transit System are within ± 5 μs of U.T.C. (U.S. Naval Observatory 1978). As a consequence, the satellite is mispositioned by about 14 cm. It will be easily possible to reduce this by an order of magnitude by using the hardware clock incorporated in the Nova satellite (Ruegar & Bates 1978).

(6) The surface forces acting on the satellite must be carefully managed: the air density at 1100 km altitude is 10^{-18} to 10^{-16} g/cm^3 (*C.I.R.A.* 1972). By using 10^{-17} as a usual value, this drag level (0.1 dyn m^{-2}†) would result in an error of 100 m/day in the ephemeris if totally uncompensated. The process of fitting the orbit by least squares approximates the quadratically growing drag error with a linear function of time. As a consequence, the error remaining in the orbit is $\frac{1}{6}(100) \approx 17$ m. If, however, as we do, we model the drag (Jacchia 1965) and additionally fit (as we currently do not) a quadratic term to absorb errors in the mean (modelled) drag level, we can remove all of the quadratic term down to a level imposed by the geodetic uncertainties. (It is significant here to mention that when a major improvement was made in the geopotential model, the secular errors in the orbit also diminished (Black 1976). An orbit–frequency term (about 30 m in amplitude) associated with the quadratic phase error was simultaneously eliminated.) The principal remaining errors due to drag uncertainty are then higher (in frequency) than the orbital rate and we estimate at twice orbit frequency. Consequently, $30\ \mathrm{m}/(2 \times 13)^2 = 0.04$ m, or 4 cm.

We can repeat this type of procedure for radiation pressure. Although the force is larger, about $\frac{1}{2}$ dyn m^{-2} of satellite cross-sectional area, the force can be more accurately modelled than can drag. When most effective (Sun in the orbit plane) the force nearly cancels when averaged over a revolution, the remaining secular effect being of the order of the eccentricity. The models are good enough to give residual effects of radiation pressure well below 10 cm.

Rather than repeating Newton's analysis of radiation pressure, I should like to mention another way of compensating for surface-force effects; the so-called 'Discos' system (Staff of the Space Department 1974). This device, a free-mass accelerometer inside the satellite, activates jets to counteract surface forces. An in-orbit experiment showed that the uncompensated surface forces were less than 10^{-11} g. The requirement for real-time positioning, and the attendant necessity for orbit prediction, provided the incentive for the 'Discos' device.

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† 1 dyn m^{-2} = 10^{-5} Nm^{-2}.

Geodetic limits

To achieve an accuracy of a few tens of centimetres, we have to extrapolate the current geopotential models beyond our existing knowlege. Currently (1978) the r.m.s. fit of the ephemeris to 48 h of data is typically 3–6 m. In the last 15 years, this value has decreased by about two orders of magnitude, about a factor of 10 in the last decade (see Black 1968). We are currently making rapid progress. The Geos-3 altimeter (Hofmeister 1973; Hofmeister *et al.* 1976) has improved our knowledge of mean sea surface topography from an uncertainty of 5 m to one of 1 m and improved (decreased) the scale size of known detail from 1000 km to less than 100 km. (Brace 1977; Anderle 1978). (See figure 9 for a comparison of the sea surface topography 'before' and 'after' Geos-3.) The 'footprint' of the Geos-3 altimeter, the area illuminated on the surface, is 3.6 km in diameter; consequently, the instrument is capable of greater detail than has yet been published. Geos-3 is still producing data on the ocean topography and its altimeter has provided a dramatic increase in our knowledge of the geoid. The Seasat–1 altimeter represents a further development of this technology; see table 2 (MacArthur 1976). With Seasat and its 10 cm

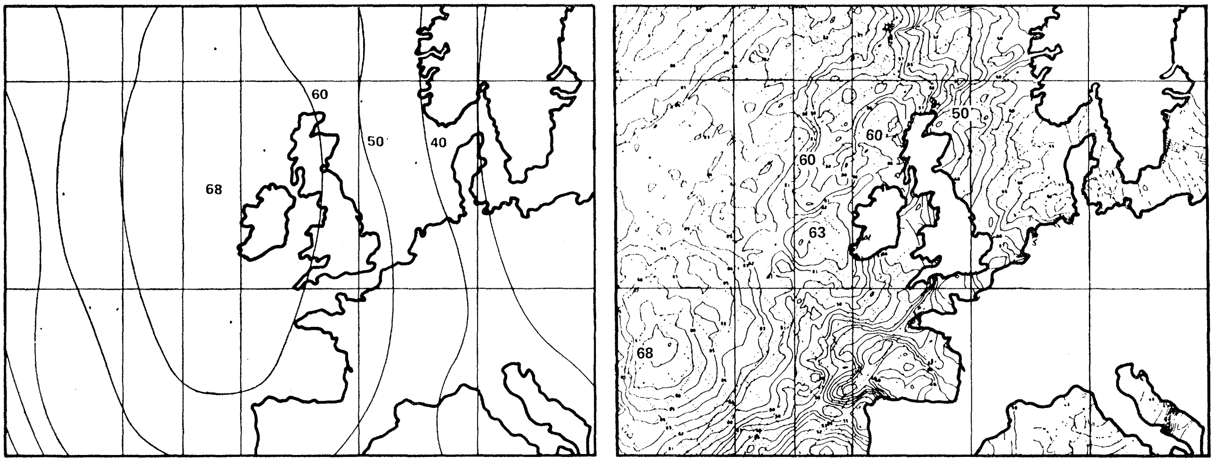

FIGURE 9. Contribution of the Geos-3 satellite altimetry to sea surface topography. The figure on the left is 'before' (*ca.* 1968) and that on the right is 'after' (*ca.* 1977) using the altimeter data. (From Brace (1977).)

TABLE 2. A COMPARISON OF ALTIMETER CHARACTERISTICS

	Geos-3 (intensive mode)	Seasat-1
mean satellite altitude/km	841	800
antenna beamwidth/deg	2.6	1.6
frequency/GHz	13.9	13.5
peak r.f. power/kW	2	2
average r.f. power/W	0.24	6.5
pulsewidth (uncompressed)/µs	1	3.2
pulsewidth (compressed)/ns	12.5	3.125
repetition frequency/Hz	100	1020
'footprint' diameter/km	3.6	1.7
altitude precision (r.m.s.)/cm	< 50	< 10
status	launched 9 April 1975	launched 26 June 1978

Source: MacArthur (1976). *Note:* Seasat-1 failed on 10 October 1978, and the Geos-3 altimeter expired on 19 September 1978.

resolution over a distance of a few kilometres, it should be possible to make corrections to the measurements (of the sea surface) for currents and tides. This is a necessary part of extending geopotential model improvements below the 1 m precision level.

The Geos-3 altimetry data are currently being used to refine geopotential models (Lerch *et al.* 1978). The next step will be to add fine-scale geopotential information obtained from satellite-to-satellite tracking (Schwartz 1970; Vonbun 1976; Roucher *et al.* 1977; Pisacane 1978).

The impression is that appreciable improvements in geopotential models are now possible and that there are no physical limitations to this process until we are well below 1 m. Certainly, below 10 cm, limits are imposed by continental drift. The time dependence of site coordinates caused by the Earth tides will have to be introduced. These displacements, mostly vertical, are of the order of 25 cm in the mid-latitudes and 50 cm near the equator (Kuo *et al.* 1970).

Error summary of an improved Transit System

Newton places the root sum square of these various error sources (except the geodetic error) at 18 cm, with drag (10 cm), uncertainty in the water vapour correction (12 cm) and instrumentation sources (9 cm) accounting for nearly all of it.

It is clear, if the numbers are accepted as an indication only, that the system is capable of (single pass) accuracy in the neighbourhood of a few tens of centimetres. The improved precision (available from multiple passes or translocation or the short-arc technique) will then reduce this to the global limit imposed by continental drift. Subsequently, if no unexpected biases intrude, the data population available for studying continental drift (and earthquake displacement fields) will appreciably increase.

For a while at least, uncertainty in the geopotential model will be the controlling error source and then, perhaps, be surplanted by third order ionospheric errors. Be that as it may, the geodetic error is destined to diminish appreciably over the next several years as a consequence of data sources that are already available or planned.

The immediate future of the Transit System

There are currently about 4000 users of the Transit System, about 700 military and the remainder civilian. The estimated investment in Transit navigation equipment is $108 M (Stansell 1978*a*, *b*). There are still 12 satellites (and 9 launch vehicles) in storage awaiting some compelling need to launch them.

If and when the U.S. Navy navigational needs currently satisfied by Transit are considered for reassignment, then the earliest that this could occur is 1985. It would then require at least 4 years to re-equip the existing ships. This would, in turn, assure Navy maintenance of the system at least until 1989 or 1990.

Several significant improvements in the system are planned for 1979.

In 1969 we began a low-level effort to rewrite the orbit determination software; the extant version was finished in 1963 and was written in assembly language for the 7094 computer.

With the announcement in the late 1960s of a new generation of computer hardware together with a computer language and system having great power (PL/1), it was clearly time to rewrite the software. We worked for several years and then put the effort aside until 1975. It was completed in early 1978 and will be implemented in 1979 on an IBM 360/65 computer at the Navy Astronautics Group at Point Mugu, California. This new program, called the Orbit

Determination Program, considerably eases the tedium associated with maintaining the system. In rewriting the program, we found no significant errors in the old one. It would have been very difficult to extend the older version to sub-metre precision because of a word-length limitation in the 7094. The IBM 7094-II computer used in the system since 1964 has been a paradigm of reliability.

Three satellites of a new design (Nova) are currently in production. One of the three is scheduled for launching in autumn 1979. Characteristics of this satellite have been previously published (Black 1976; Staff 1975). None of these changes will require any user to modify his software or hardware.

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I am indebted to S. C. Dillon, W. L. Ebert, B. B. Holland, M. M. Feen, and R. B. Kershner for their assistance in preparing this paper.

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Discussion

N. A. G. Leppard (*Directorate of Military Survey, Feltham, Middlesex, U.K.*). With the launching of the Nova satellites, will the number of satellites available for navigation and geodetic use be increased, or will the present constellation of five or six satellites be maintained?

H. D. Black. The current position is that a constellation of (at least) four satellites will be maintained at least until 1990.

J. C. Blankenburgh (*Continental Shelf Institute, Trondheim, Norway*). The first-order correction model for ionospheric refraction is used on the assumption of normal conditions, that is when both wave paths cross the same parallel ionospheric layers of equal electron content. Mr Black stated that the correction is modelled best in summer and during the night. Is this also true for the northernmost areas where the ionospheric activity is at maximum during the winter night?

H. D. Black. For a number of years we had a tracking station in Fairbanks, Alaska and during that period found the first-order ionospheric correction to be entirely satisfactory. Currently, the accuracy of the system is much improved and I would not be surprised if third-order ionospheric effects were an important error source, particularly at high latitudes.

P. G. Sluiter (*c/o Shell EP/*12, *P.O. Box* 162, *The Hague, Netherlands*). From graphs that I have prepared of the values of the major broadcast orbit parameter over the past 10 years, the changes in the semimajor axis showed some striking discontinuities. During 1968–70 the satellites were falling at the rate of nearly 1 m/day. Beginning 1971 this decreased suddenly and reached about 0.25 m/day in 1976. At the end of 1977 this increased again sharply to about 1 m and more per day. Would Dr Black say that this is also due to the cycle in solar activity that he referred to?

H. D. Black. The Chairman may like to reply.

D. G. King-Hele, F.R.S. This is certainly so. Satellites in low orbits have been falling back to Earth at a much increased frequency during the past year. The satellite decay rates are directly proportional to the upper atmospheric air density, which in turn increases with the solar u.v. flux, and we are now approaching the maximum of the 11 year solar cycle.